OGLEDNI PRIMJERAK, MAT 2,

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1) Provjerite da je $y = \frac{1}{2(\ln x+1)}$ rješenje dif. jdn.

$xy' - y(2y\ln x - 1) = 0$

Rj

$y' = -\frac{1}{2(\ln x+1)^2} \cdot \frac{1}{x}$ pa imamo

$xy' - y(2y\ln x - 1) = -\frac{1}{2(\ln x+1)^2} - \frac{1}{2(\ln x+1)}\left(\frac{\ln x}{\ln x+1} - 1\right) =$

$= -\frac{1}{2(\ln x+1)^2} - \frac{1}{2(\ln x+1)^2} = 0$

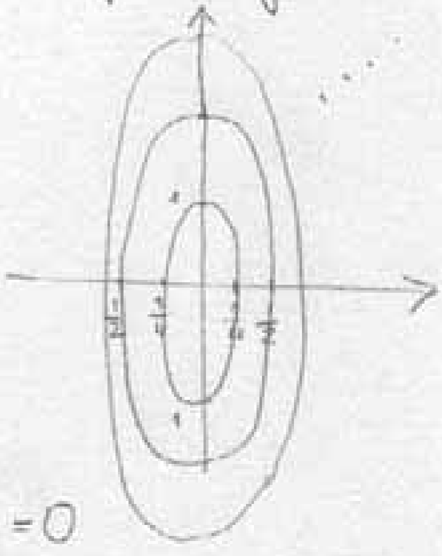
2) Odredite dif. jednadžbu familije int. krivulja $4x^2 + y^2 = C^2$ i skicirajte tu familiju

Rj

$4x^2 + y^2 = C^2$

$\boxed{4x^2 + 2yy' = 0}$

$\frac{x^2}{\frac{C^2}{4}} + \frac{y^2}{C^2} = 1$ elipse $\Rightarrow$



3) Riješiti C-problem: $y\,dx + \operatorname{ctg} x\,dy = 0$

$y\left(\frac{\pi}{3}\right) = -1$

Rj

imamo $y + \operatorname{ctg} x \cdot y' = 0$

$\operatorname{ctg} x \cdot y' = -y$

$\frac{dy}{y} = -\frac{dx}{\operatorname{ctg} x} \Big/ \int$

$\ln y = -\int \frac{\sin x}{\cos x}\,dx = \ln(\cos x) + \ln C$

$y = C \cdot \cos x$

$y\left(\frac{\pi}{3}\right) = -1 \Rightarrow -1 = C \cdot \cos\frac{\pi}{3}$

$-1 = C \cdot \frac{1}{2}$

$C = -2$

$y = -2\cos x$

4) Riješite C-problem $y' - y\,tg\,x = \frac{1}{\cos x}$ $y(0)=0$

Rj pro homogena:

$$y' - y\,tg\,x = 0$$
$$y' = y\,tg\,x$$
$$\frac{dy}{y} = tg\,x\,dx \quad / \int$$
$$\ln y = -\ln(\cos x) + \ln C$$
$$y = \frac{C}{\cos x}$$

nehomogena: $y = \frac{C(x)}{\cos x}$

$$y' = \frac{C'(x)\cos x + C(x)\sin x}{\cos^2 x}$$

$$\frac{C'(x)}{\cos x} + C(x)\frac{\sin x}{\cos^2 x} - \frac{C(x)}{\cos x}\cdot\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\frac{C'(x)}{\cos x} = \frac{1}{\cos x} \Rightarrow C'(x) = 1$$
$$C(x) = x + D$$
$$y = \frac{x+D}{\cos x}$$

$$y(0) = 0 = \frac{0+D}{\cos 0} = D \Rightarrow y = \frac{x}{\cos x}$$

5) Riješite jdu $y'' - y = \sin x$

Rj $y = y_H + y_P$ $\quad y_H$: $\lambda^2 - 1 = 0$

$$\lambda = \pm 1$$
$$y_H = C_1 e^{x} + C_2 e^{-x}$$

y_P: $f(x) = \sin x$

$a = 0,\ b = 1,\ P_n(x) = 0,\ Q_m(x) = 1$, $a + bi = i$ nije korijen

$$y_P = A\cos x + B\sin x$$
$$y_P' = -A\sin x + B\cos x \qquad y_P'' = -A\cos x - B\sin x$$

$$-A\cos x - B\sin x - A\cos x - B\sin x = \sin x \qquad A = 0 \quad B = -\frac{1}{2}$$
$$-2A\cos x - 2B\sin x = \sin x$$

$$y_P = -\frac{1}{2}\sin x$$
$$y = C_1 e^{x} + C_2 e^{-x} - \frac{1}{2}\sin x$$